The Rabbani-Kloek Elimination
A Method for Answering Multiple Choice Questions on AP Exams

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I recall a winter's day in 2002 as I was sitting in my fourth period AP Calculus class: my mind was wandering and that frisky snow-draped tree clinked incessantly against the second story Plexiglas window in room two-hundred and something. Our teacher was passing back our exams (on, say, L'Hôpital's Rule) and my classmates and I were doing our best to maintain our composure.

My exam plopped onto my desk as if it has fallen from divinity, and suddenly I hear the ferocious rustling of pages nearby. Paul Kloek, first row, three seats to the left of the overhead projector and, incidentally, directly to my right, began to express his frustration over a particularly convoluted problem on the exam.

"I thought it tends to zero," he says to me.

Our teacher chimes in, "Just because it's complicated doesn't mean it tends to zero."

Kloek wittingly and self-effacingly rebutted with this: "Well, I just figured that that couldn't possibly be the answer, and that it must tend to zero or something."

Our teacher let out an amused guffaw and declared, "Kloek's Rule: 'when an expression becomes too hard to solve, it tends to zero.' "

Kloek smiled in a sort of sarcastically proud way.

Of course, one does not need a mathematician to point out the flaws in such a "rule." Aside from being mathematically unsound, Kloek's Rule requires a subjective judgment of the difficulty of solving the expression in question.

But Kloek's reasoning had me thinking. In reality, of course, a mathematical expression can be as complicated as Schrödinger's equation, but on a high school calculus exam, such sophistication is unlikely. In fact, we often find that the solutions to such exams turn out to be clean and tailored.

Several weeks later, Kloek and I were in class looking over a practice AP exam, and something profound occurred to us as we stared at this one elusive multiple choice question.

"I think it's C," Kloek said.

"I agree, though I didn't really solve it."

"Nor did I."

And at that moment, I knew that Kloek had solved the problem the same way I had: by a systematic process of elimination that has its roots in the inner workings of the exam writers. We had tacitly developed what later came to be called the Rabbani-Kloek Elimination, a method for finding the answer to multiple choice questions on AP exams, specifically those involving symbolic and numerical choices. No method, of course, is more rewarding than actually solving the problem, but the Elimination allows one to answer a certain class of questions without any knowledge of the course material or any consideration of the problem statement.

During my subsequent years in high school, after Kloek had graduated, I taught this method to eager AP Calculus students, many of whom later informed me that they owed a good portion of their exam grades to the Elimination.

These positive reports ignited a tremendous enthusiasm for the Elimination, to which I now respond with this explanatory pamphlet. Below is the method delineated and applied. I sincerely hope that the Rabbani-Kloek Elimination will continue to help AP students until the College Board is made aware of and remedies this gaping hole that makes the Rabbani-Kloek Elimination possible.

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The Rabbani-Kloek Elimination

For multiple choice questions on AP exams involving symbolic and numerical answers that are readily grouped according to common features, one may find the answer with about 80% certainty by applying this process:

1. Divide the five answers into two groups (with cardinalities of four and one or three and two) based on common features.
2. Eliminate the answers that fall into the minority group.
3. Repeat the above steps, always considering all five answer choices, until only one answer choice survives.
4. Select the surviving answer choice.

Applications of the Elimination

Let us consider realistic multiple choice AP questions. The question that appears below has been readily available in the downloadable "Course Description" for AP Calculus BC on the College Board website in previous years[1]

1. The area of one loop of the graph of the polar equation $r = 2\sin(3\theta)$ is given by which of the following expressions?
   (a) $4\int_0^{\frac{\pi}{3}} \sin^2(3\theta)d\theta$
   (b) $2\int_0^{\frac{\pi}{3}} \sin(3\theta)d\theta$
   (c) $2\int_0^{\frac{\pi}{3}} \sin^2(3\theta)d\theta$
   (d) $2\int_0^{\frac{2\pi}{3}} \sin^2(3\theta)d\theta$
   (e) $2\int_0^{\frac{2\pi}{3}} \sin(3\theta)d\theta$

We ignore the question and look solely at the answer choices: these choices are easily separable. At a quick glance, we realize that all but one of the answer choices has an integral that is multiplied by 2. Since choice (A) is in the minority, we eliminate it. We then observe all five answer choices again and consider the limits of integration, noticing that the upper limit on three of the choices is $\pi/3$ while it is $2\pi/3$ on the other two. Since the latter are in the minority, we eliminate them. This leaves choices (B) and (C). We notice that these choices differ only by the squaring of the sine. Considering all the choices again, we find that three of them contain a sine squared term while only two do not. We eliminate the latter, leaving us with one answer choice: (C). Solving this problem as the College Board meant us to do it indeed reveals (C) as the correct answer.

Next, we consider another example from the College Board website that involves numerical answers. Through this example, we demonstrate that although the Rabbani-Kloek Elimination is more difficult to apply to the class of numerical answer choices, it is still possible and often more convenient for the unsure test-taker.

2. The line perpendicular to the tangent of the curve represented by the equation $y = x^2 + 6x + 4$ at the point $(-2, -4)$ also intersects the curve at $x =$
   (a) -6

[1] The problems used here are a few years old as they correspond to the time when this article was conceived. The more recent Course Description is available at http://www.collegeboard.com/student/testing/ap/sub_calbc.html?calcbc.

(b) $-\frac{9}{2}$
(c) $-\frac{7}{2}$
(d) -3
(e) $-\frac{1}{2}$

We first notice that three of the five choices involve fractions while two do not. We eliminate the latter, leaving (B), (C), and (E). At a second glance, we see that multiples of 3 are prevalent in these answer choices, and those answers that do not include a multiple of three contain prime numerators. Namely, (A), (B), and (D) involve multiples of three and (C) and (E) do not. We therefore eliminate the latter, leaving us with only one answer choice: (B). Solving this problem using differentiation does in fact yield (B).

In this example and in many other applications, the criteria for separation may seem somewhat arbitrary and forced. Though this may be the case, the Rabbani-Kloek Elimination requires the test-taker to become intimate with the answer choices and to attain a sort of basic understanding of the motivation behind them. We could also have separated the answer choices into those involving numerators that are greater than 5, leading to the elimination of (D) and (E). This criterion, however, is less robust than the divisibility by 3, for it admits no real mathematical thought. When we consider divisibility by 3, we do so with the understanding that some factor of three may have played out in the solution. However arbitrary this seems, it is a considerably sounder criterion in the absence of striking differences in the answer choices.

In thinking about what criteria to use, it is expedient to consider the form in which the answer choices are presented. In our first example, we encounter integrals, so we may want to examine various properties of the integral; in our case, we examined the upper limit of integration. In the example with the fractions, we examined divisibility; however, if the answer choices were to have been presented in decimal form, we would not speculate that divisibility is an essential facet of the solution. In such a case, we could consider properties of the answer choices arising from the fact that they are decimals, such as commonalities in the post- or pre-decimal sections of the answer choice.

For good measure, we consider one final question, drawn, again, from the sample questions on the Course Description:

3. If $\frac{d}{dx}f(x) = g(x)$ and if $h(x) = x^2$, then $\frac{d}{dx}f(h(x)) =$

   (a) $g(x^2)$
   (b) $2xg(x)$
   (c) $g'(x)$
   (d) $2xg(x^2)$
   (e) $x^2g(x^2)$

We notice that three choices have an argument of x^2 in the function g. We eliminate the minority choices (B) and (C). Further, we see that three choices admit coefficients of the function g that contain x, while two do not. We eliminate (A) and (C). At this point, we are left with two viable options: (D) and (E). We can impose one final, less robust criterion: the presence of a coefficient in the second order of x. This leaves (D) as the solution, which is confirmed by solving this problem mathematically.

In this particular example, we may have been tempted to impose the criterion of the $2x$ term, which would have placed our correct answer in the minority. However, this would leave (A), (C), and (E) in our majority group, where (E) would be sufficiently different from (A) and (C) in its coefficient that it would be inappropriate to group (E) alongside them. This phenomenon leads to an important corollary to the Rabbani-Kloek Elimination: we must leave criteria that result in the creation of more than two groups as a last resort, to be applied only at the very end. Indeed, our final criterion was the very coefficient of the function. In essence, we created three groups (A) and (C); (B) and (D); and (E). Since our final group has the least number of constituents, we eliminate it. This method leads to the correct answer in our case. Nevertheless, this application of the Elimination is less robust than binary grouping. In some cases, the Elimination leaves more than one answer after all reasonable criteria are exhausted. In such situations, it is beneficial to randomly answer the question from among the surviving choices despite the penalty for an incorrect answer, for the expected return is still positive as long as one incorrect answer is eliminated.

Cursory anecdotal evidence seems to suggest that the Rabbani-Kloek Elimination can yield the correct answer as often as 80% of the time for AP Calculus multiple choice questions where it applies.[2] The theory governing the Elimination delves into the thought process of the exam's authors; namely, we believe that the method is effective because the exam writers endeavor to make the question as difficult as possible to answer without solving the problem completely. For example, if we were solving question (1) above and part way through we realize that the upper limit of integration must be $\pi/3$, we would only have a $1/3$ chance of answering it correctly without continuing on to the final solution. Because the Elimination reflects the general psychology of the College Board, it has also been shown to be exceedingly effective on other AP exams in technical subjects.

All said and done, it is important to stress that the Rabbani-Kloek Elimination is a last resort. If mathematically computed solutions conflict with the Rabbani-Kloek solution, the former must take priority, and the test-taker should disregard the latter solution without loss of confidence. Always keep in mind that the Elimination is intended to assist, not to impair.

[2]More rigorous tests of statistical significance corroborating the efficacy of the Elimination are still wanting. We may regard the Elimination as a sort of logical conjecture in the absence of such a formal study.